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Contact Stress Analysis of Stainless Steel Spur Gears using Finite Element Analysis and Comparison with Theoretical Results using Hertz Theory

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Abstract
Gears or toothed wheels form a positive drive for power transmission system in precision machines wherein a definite velocity ratio is needed. Despite having high cost, complicated manufacturing, need of precise alignment of shafts and lubrication, the gear drives are preferred over other power transmission drives. One of the important reasons of preference being that of efficiency which is very high in gear drives, even upto 99 per cent in case of spur gears. Spur gears are the simplest of the gear drives having teeth cut parallel to the axis of the shaft. Herein, we report the contact stress analysis of Stainless Steel spur gears by theoretical method using Hertz equations and by Finite Element Analysis using FEA software ANSYS 14.0 Workbench. The spur gear is sketched and modelled in ANSYS Design Modeller and the contact stress analysis is done in Mechanical ANSYS Multiphysics. When compared, the results of both theoretical method and FEA show a good degree of agreement with each other.
Keywords: Spur Gear, Contact Stress, Hertz Equations, ANSYS 14.0 Workbench, Finite Element Analysis.

I. Introduction

The spur gears when in action, are subjected to several stresses but out of all, two types of stresses viz., bending stress and contact stress are important from the design point of view. The bending stresses are theoretically analysed by Lewis equation and contact stresses by Hertz equation. Since spur gears have complicated geometry, a need arises for improved analysis using numerical methods which provide more accurate solutions than the theoretical methods. Finite Element Analysis is one such method which has been extensively used in analysis of components used in various mechanical systems.

The Finite element analysis of spur gears has been reported by some researchers in the recent past. Bharat Gupta et al., [1] reported the contact stress analysis of spur gears and concluded that the contact stress forms the basis of resisting the gear pitting failure. They also concluded that module is an important parameter for gear design and showed that for the transmission of large power and minimization of contact stress, a spur gear with higher module must be preferred. T. Shoba Rani et al., [2] have performed finite element analysis on spur gear using different materials viz., nylon, cast iron and polycarbonate. They observed that in order to get good efficiency, life and less noise cast iron gears can be replaced by nylon gears because of the fact that the deflection of cast iron is more than that of nylon. Sushil Kumar Tiwari et al., [3] analysed the contact stress and bending stress of involute spur gear teeth by FEA and compared the same with Hertz equation, Lewis equation and AGMA/ANSI equations. The FEA results showed a good degree of agreement with the theoretical results. Vivek Karaveer et al., [4] investigated the contact stress analysis of spur gears and showed that the results of Hertz equation and FEA are comparable. The materials used in their study were, cast iron and steel. Moreover the contact stress was determined during the transmission of torque of 15000 lb-in or 1694.7725 Nm [5] using finite element analysis. M. Raja Roy et al., [6] also reported the contact pressure analysis of spur gear using FEA and concluded that with increase in module the maximum allowable contact stress on involute pair of spur gear teeth decreases.

In the present study, we report the contact stress analysis of 14.5 degree full depth involute stainless steel spur gears during the transmission of power of 10kW by theoretical method using Hertz theory and by FEA using ANSYS Workbench 14.0. A general mathematical model is proposed for evaluating the contact stress in stainless steel spur gears of equal geometry in mesh using hertz equations. The theoretical results are compared with the results of FEA.

II. Material properties

The material chosen for the study is Stainless Steel. Table 1 shows the properties of Stainless Steel as presented in ANSYS Workbench 14.0 engineering data sources.

S. No.	Property	Value	Unit
1.	Density	7750	Kg/m^3
2.	Coefficient of thermal expansion	1.7E-05	C^{-1}
3.	Young's modulus	1.93E+11	Pa
4.	Poison's ratio	0.31	
5.	Bulk modulus	1.693E+11	Pa
6.	Shear modulus	7.3664E+10	Pa
7.	Tensile yield strength	2.07E+08	Pa
8.	Compressive yield strength	2.07E+08	Pa
9.	Tensile ultimate Strength	5.86E+08	Pa
10.	Compressive ultimate strength	0	Pa

Table 1: Properties of Stainless Steel

III. Modelling of Spur Gear

The Spur gear is sketched and modelled in ANSYS Design Modeller. The dimensions of the gear set are given in Table 2. The values are same for both gears in assembly.

S. No.	Description	Units	Symbol	Value
1	No. of Teeth on Pinion		z_p	32
2	No. of Teeth on Gear		z_g	32
3	Pressure Angle	∘	ϕ	14.5
4	Module	mm	m	6
5	Addendum	mm	h_a	m = 6
6	Dedendum	mm	h_d	1.157m = 6.942
7	Pitch Circle Diameter	mm	d_p	mz = 6×32 = 192
8	Pitch Circle Radius	mm	r_p	96
8	Base Circle Diameter	mm	d_b	$d_p\cos\phi$ = 192×0.96815 =185.884
9	Addendum Circle Diameter	mm	d_a	dp + 2m = 204
10	Dedendum Circle Diameter	mm	d_d	dp- $(2 + \pi/z)\times$m = 179.410
11	Face Width	mm	b	10m = 10×6 = 60
12	Fillet Radius	mm	r_p	0.4m = 0.4×6 = 2.4
13	Shaft Radius	mm	r_s	48

Table 2: Dimensions of the gear set

The following figures show the 2D sketch of Spur gear and 3D model of Spur gear respectively.

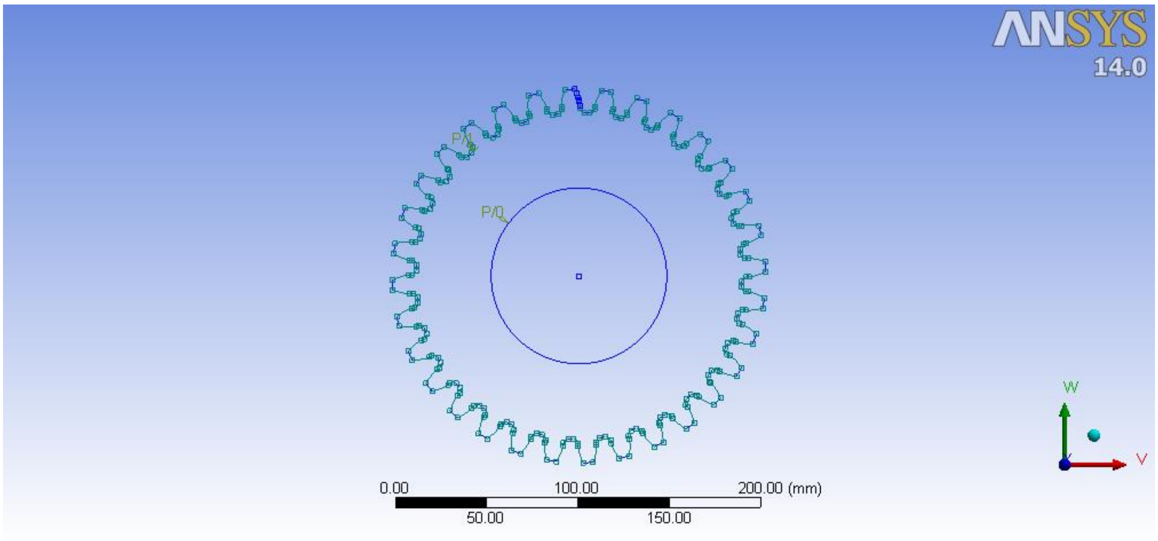

Figure 1: Sketching of Spur Gear

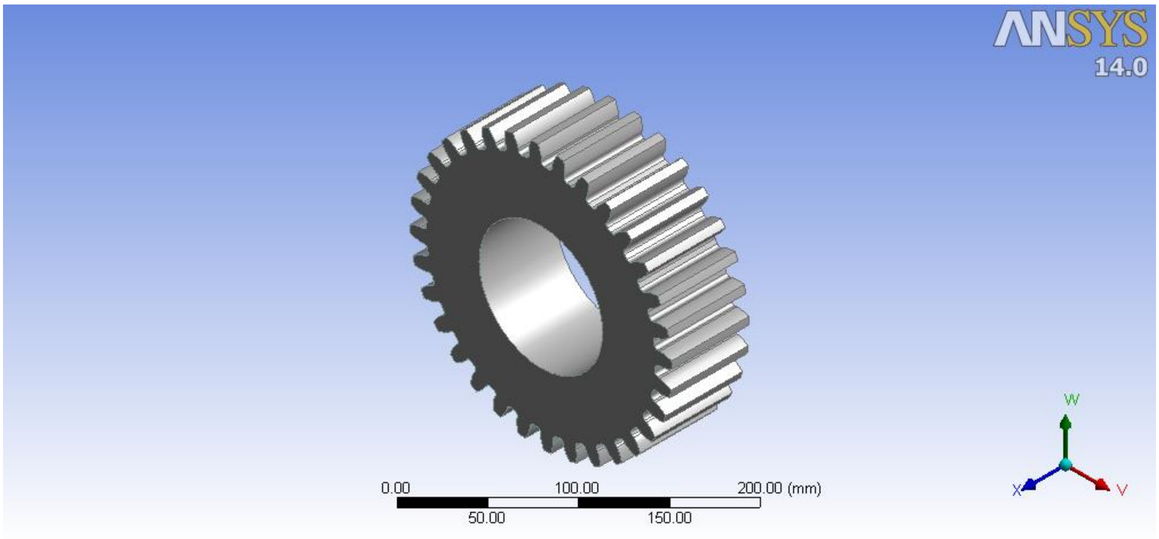

Figure 2: Three Dimensional Model of Spur Gear

IV. Theoretical Analysis of Contact Stress using Hertz Equations:

Earle Buckingham (1926) used the Hertz theory to determine the contact stress between a pair of teeth while transmitting power by treating the pair of teeth in contact as cylinders of radii equal to the radii of curvature of the mating involutes at the pitch point. According to Hertz theory as given in [7], when two cylinders are pressed together, the contact stress is given by,

$$\sigma_c = {}^{2P}/_{\pi BL} \qquad \text{(i)}$$

$$\text{and } B = \sqrt{\left[\frac{2P\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}{\pi l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}\right]} \qquad \text{(ii)}$$

Where,
σ_c= maximum value of contact stress (N/mm^2)
P= force pressing the two cylinders together (N)
B = half width of deformation (mm)
L= axial length of cylinders (mm)
d_1,d_2 = diameters of two cylinders (mm)
E_1,E_2 = modulii of elasticity of two cylinder materials (N/mm^2)
μ_1, μ_2= poisson's ratio of the two cylinder materials (unitless)

Substituting the value of half width of deformation B, in equation (i) & squaring both sides we get,

$$\sigma_c^2 = \frac{1}{\pi}\left(\frac{P}{L}\right)\left[\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}\right] \quad \text{(iii)}$$

If we treat the material of both the cylinders as same, then the modulii of elasticity and poisson's ratio will be equal. Therefore substituting $E_1 = E_2 = E$ and $\mu_1 = \mu_2 = \mu$ in equation (iii) we get,

$$\sigma_c^2 = \frac{1}{2\pi}\left(\frac{P}{L}\right)\left[\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1-\mu}{E}\right)}\right] \quad \text{(iv)}$$

Now applying this equation to a pair of spur gear teeth in contact, we need to replace the radii r_1 & r_2 by the radii of curvature at the pitch point [7].

Therefore we have,

$$r_1 = \frac{d_{pp}Sin\phi}{2} \text{ and } r_2 = \frac{d_{pg}Sin\phi}{2}$$

where, d_{pp} = pitch circle diameter of pinion
d_{pg} = pitch circle diameter of gear

Now, since the pinion and gear have equal geometry in all respects as given in table 2 in section 3 above, therefore we have,

$$d_{pp} = d_{pg} = d_p$$

Therefore we have,

$$r_1 = \frac{d_pSin\phi}{2} \text{ and } r_2 = \frac{d_pSin\phi}{2}$$

$$\Rightarrow \; r_1 = r_2 = r = \frac{d_pSin\phi}{2} \quad \text{(v)}$$

Substituting $r_1 = r_2 = r$ in equation (iv) we get,

$$\sigma_c^2 = \frac{1}{\pi}\left(\frac{P}{L}\right)\left[\frac{\left(\frac{1}{r}\right)}{\left(\frac{1-\mu}{E}\right)}\right]$$

$$\Rightarrow \sigma_c^2 = \frac{1}{\pi(1-\mu)}\left(\frac{P}{L}\right)\left[\frac{\left(\frac{1}{r}\right)}{\left(\frac{1}{E}\right)}\right]$$

$$\Rightarrow \; \sigma_c^2 = \frac{1}{\pi(1-\mu)}\left(\frac{PE}{Lr}\right) \quad \text{(vi)}$$

If we treat the material of both the mating pinion and gear as stainless steel, then for stainless steel, from table 1 poisson's ratio $\mu = 0.31$. Substituting this value of μ in equation (vi) and solving we have,

$$\sigma_c = 0.6792\left(\frac{PE}{Lr}\right)^{\frac{1}{2}} \quad \text{(vii)}$$

Now for stainless steel, again from table 1 modulus of elasticity $E = 193000MPa$. Substituting this value of E in equation (vii), we have,

$$\sigma_c = 0.6792\left(\frac{P \times 193000}{Lr}\right)^{\frac{1}{2}}$$

$$\Rightarrow \sigma_c = 0.6792 \times 439.3177\left(\frac{P}{Lr}\right)^{\frac{1}{2}}$$

$$\Rightarrow \sigma_c = 298.386\left(\frac{P}{Lr}\right)^{\frac{1}{2}} \quad \text{(viii)}$$

Now from equation (v), we have

$$r = \frac{d_pSin\phi}{2}$$

$$\Rightarrow r = \frac{d_pSin\phi}{2} = r_pSin\phi$$

Where, r_p is the radius of pitch circle for both pinion and gear.

Substituting this value of r in equation (viii) we get,

$$\sigma_c = 298.386\left(\frac{P}{Lr_pSin\phi}\right)^{\frac{1}{2}} \quad \text{(ix)}$$

Now, $P = \frac{P_t}{Cos\phi}$, where, P_t is the tangential component of the resultant force P between two meshing teeth. Substituting this valve of P in equation (ix) we get,

$$\sigma_c = 298.386\left(\frac{P_t}{Lr_pSin\phi Cos\phi}\right)^{\frac{1}{2}} \quad \text{(x)}$$

Also, the axial length L is equal to the face width b of spur gears, therefore replacing L by b in equation (x) we get,

$$\sigma_c = 298.386\left(\frac{P_t}{br_pSin\phi Cos\phi}\right)^{\frac{1}{2}} \quad \text{(xi)}$$

Equation (xi) is the general mathematical model for evaluating the contact stress for a pair of stainless steel spur gear teeth in contact, for the equal geometry & dimensions of pinion and gear in mesh.

Considering the power to be transmitted $P = 10kW$ and the speed of rotation of the pinion $n_p = 1500rpm$, we can evaluate the tangential component of force P_t.

Power to be transmitted in kW from [7] is given by,

$$P = \frac{2\pi n_p T}{60 \times 10^6}$$

Where, T is transmitted torque in N-mm

$$\Rightarrow 10 = \frac{2\pi \times 1500 \times T}{60 \times 10^6}$$

$$\Rightarrow T = \frac{10 \times 60 \times 10^6}{2\pi \times 1500}$$

$$\Rightarrow T = 63661.977 Nmm$$

Also,

$$P_t = \frac{2T}{d_p}$$

From table 2, $d_p = 192mm$

Therefore we have,

$$P_t = \frac{2 \times 63661.977}{192}$$

$$\Rightarrow P_t = 663.1456\ N$$

Again from table 2, $b = 60mm$, $\phi = 14.5°$ and $r_p = 96mm$

Substituting these values in equation (xi) we have

$$\sigma_c = 298.386 \left(\frac{P_t}{br_p Sin\phi Cos\phi}\right)^{\frac{1}{2}}$$

$$\Rightarrow \sigma_c = 298.386 \left(\frac{663.1456}{60 \times 96 \times Sin14.5° \times Cos14.5°}\right)^{\frac{1}{2}}$$

$$\Rightarrow \sigma_c = 298.386 \left(\frac{663.1456}{5760 \times 0.2424}\right)^{\frac{1}{2}}$$

$$\Rightarrow \sigma_c = 298.386 \left(\frac{663.1456}{1396.224}\right)^{\frac{1}{2}}$$

$$\Rightarrow \sigma_c = 298.386(0.474956)^{\frac{1}{2}}$$

$$\Rightarrow \sigma_c = 298.386 \times 0.68917$$

$$\Rightarrow \sigma_c = 205.6387 MPa$$

Allowable maximum contact stress,

$$\sigma_{ca} = \frac{\sigma_c}{S.F.}$$

Where S.F. is the safety factor.

The Safety factor can be taken from ANSYS results or from other safety factor tables. In the present study we have taken safety factor from ANSYS results as is given in Figure 6 in section 6. We prefer to take minimum safety factor from ANSYS results so as to get more accurate value of allowable maximum contact stress. As is clearly indicated in figure 6 the minimum safety factor is 2.7356.

$$\Rightarrow \sigma_{ca} = \frac{205.6387}{2.7356}$$

$$\Rightarrow \sigma_{ca} = 75.171 MPa$$

Again from equation (xi) we have,

$$\sigma_c = 298.386 \left(\frac{P_t}{br_p Sin\phi Cos\phi}\right)^{\frac{1}{2}}$$

$$\Rightarrow P_t = br_p Sin\phi Cos\phi \left(\frac{\sigma_c}{298.386}\right)^2 \qquad \text{(xii)}$$

Replacing σ_c by the maximum allowable hertz stress σ_{hlim} for the material in equation (xii) we get the tooth surface strength of the pinion P_{ts}. [8]

$$\Rightarrow P_{ts} = br_p Sin\phi Cos\phi \left(\frac{\sigma_{hlim}}{298.386}\right)^2 \qquad \text{(xiii)}$$

The maximum allowable hertz stress σ_{hlim} for stainless steel spur gears is 41.3 kgf/mm^{2}which is equal to 405.015 N/mm^2.[9]

$$\Rightarrow P_{ts} = 60 \times 96 \times Sin(14.5°) \times Cos(14.5°) \times \left(\frac{405.015}{298.386}\right)^2$$

$$\Rightarrow P_{ts} = 2572.4625N \qquad \text{(xiv)}$$

For the design to be safe, the tooth surface strength P_{ts} must be greater than the dynamic load on gear tooth P_d.[8]

The dynamic load on gear tooth from [7] is given by,

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{Ceb + P_t}} \qquad \text{(xv)}$$

Where, v = pitch line velocity (m/s)
C =deformation factor (N/mm^2)
e =Sum of errors between two meshing teeth (mm)
b =face width of tooth (mm)
P_t = tangential component of force (N)

The deformation factor C from [7] is given by,

$$C = \frac{k}{\left[\frac{1}{E_1} + \frac{1}{E_2}\right]}$$

Where, k = constant depending on form of tooth.
E_1 =modulus of elasticity of pinion material (N/mm^2)
E_2 =modulus of elasticity of gear material (N/mm^2)

In our case, the material for both pinion and gear is stainless steel. Therefore we have

$$E_1 = E_2 = 193000MPa = 193000N/mm^2$$

Also for 14.5° full depth involute system $k = 0.107$

Therefore we have,

$$C = \frac{0.107}{\left[\frac{1}{193000} + \frac{1}{193000}\right]}$$

$$\Rightarrow C = \frac{0.107 \times 193000}{2}$$

$$\Rightarrow C = \frac{0.107 \times 193000}{2}$$

$$\Rightarrow C = 10325.5N/mm^2$$

The pitch line velocity v, from [7] is given by,

$$v = \frac{\pi d_p n}{60 \times 10^3}$$

$$\Rightarrow v = \frac{\pi \times 192 \times 1500}{60 \times 10^3}$$

$$\Rightarrow v = \frac{904778.6842}{60 \times 10^3}$$

$$\Rightarrow v = 15.0796\ m/s$$

The error e, is a function of the quality of the gear and the method of manufacturing it. There are twelve different grades of gears as listed in table 3 which form the basis of method of manufacture of gears. The table 3 also lists the tolerances for adjacent pitch error for each of the twelve gear grades [7].

Grade	***e*(microns)**
1	$0.80 + 0.06\phi$
2	$1.25 + 0.10\phi$
3	$2.00 + 0.16\phi$
4	$3.20 + 0.25\phi$
5	$5.00 + 0.40\phi$
6	$8.00 + 0.63\phi$
7	$11.00 + 0.90\phi$
8	$16.00 + 1.25\phi$
9	$22.00 + 1.80\phi$
10	$32.00 + 2.50\phi$
11	$45.00 + 3.55\phi$
12	$63.00 + 5.00\phi$

Table 3: Tolerances on the adjacent pitch.[7]

From [7] the tolerance factor ϕ is given by,

$$\phi = m + 0.25\sqrt{d_p}$$

where,
m =module (mm)
d_p = pitch circle diameter (mm)

In our case,

$$m = 6mm\ \&\ d_p = 192mm$$

Therefore we have,

$$\phi = 6 + 0.25\sqrt{192} \tag{xvi}$$

Again from [7] the error e is given by,

$$e = e_p + e_g \tag{xvii}$$

where,
e_p =error for pinion
e_g = error for gear

Since, in our case the pinion and gear are of equal geometry in all respects, therefore tolerance factor ϕ is same for both gear and pinion. Also the grades listed in table 3 from grade 1 to grade 12 are arranged in decreasing order of precision. Considering the gear and pinion to be of grade 1 which is of top precision, we have

$$e_p = e_g = 0.80 + 0.06\phi \tag{xviii}$$

From equations (xvii) & (xviii) we have,

$$e = 2e_p = 2e_g = 2(0.80 + 0.06\phi) \tag{xix}$$

From equation (xvi) we have

$$\phi = 6 + 0.25\sqrt{192}$$

Substituting this value of ϕ in equation (xix) we have,

$$e = 2\left[0.80 + 0.06\left(6 + 0.25\sqrt{192}\right)\right]$$

$$\Rightarrow e = 2.7357\mu m = 2.7357 \times 10^{-3}mm$$

Now from equation (xv) we have, the dynamic load,

$$P_d = \frac{21v(Ceb + P_t)}{21v + \sqrt{Ceb + P_t}}$$

Substituting the values of e, P_t, C, v and b in the above the equation we have,

$$P_d = \frac{21 \times 15.0796\ [(10325.5 \times 2.7357 \times 10^{-3} \times 60) + 663.1456]}{21 \times 15.0796\ + \sqrt{(10325.5 \times 2.7357 \times 10^{-3} \times 60) + 663.1456}}$$

$$\Rightarrow P_d = 2044.487N \quad \text{(xx)}$$

From (xiv) and (xx) we have,

$$P_{ts} > P_d$$

Therefore the design is safe from surface durability considerations.

V. .Finite Element Analysis Using ANSYS Workbench 14.0:

Finite Element Analysis is the practical application of the finite element method (FEM), which is used by engineers and scientists to mathematically model and numerically solve very complex structural, fluid, and multiphysics problems [10]. It is a computational tool for engineering analysis which makes use of mesh generation techniques for dividing a complex problem into small elements coupled with the use of a software program coded FEM algorithm [11]. Finite element analysis softwares such as ANSYS workbench provide a computerized approach for predicting the behaviour of the object to real-world forces, vibration, heat, fluid flow, and other physical effects. In the present study, FEA software ANSYS 14.0 Workbench has been used to determine the maximum allowable contact stress in stainless steel spur gears. Fine meshing as shown in figure 3, has been done in order to get accurate results. A moment of 63661.977 N-mm is applied in clockwise direction on the inner rim of the upper gear (figure 4).

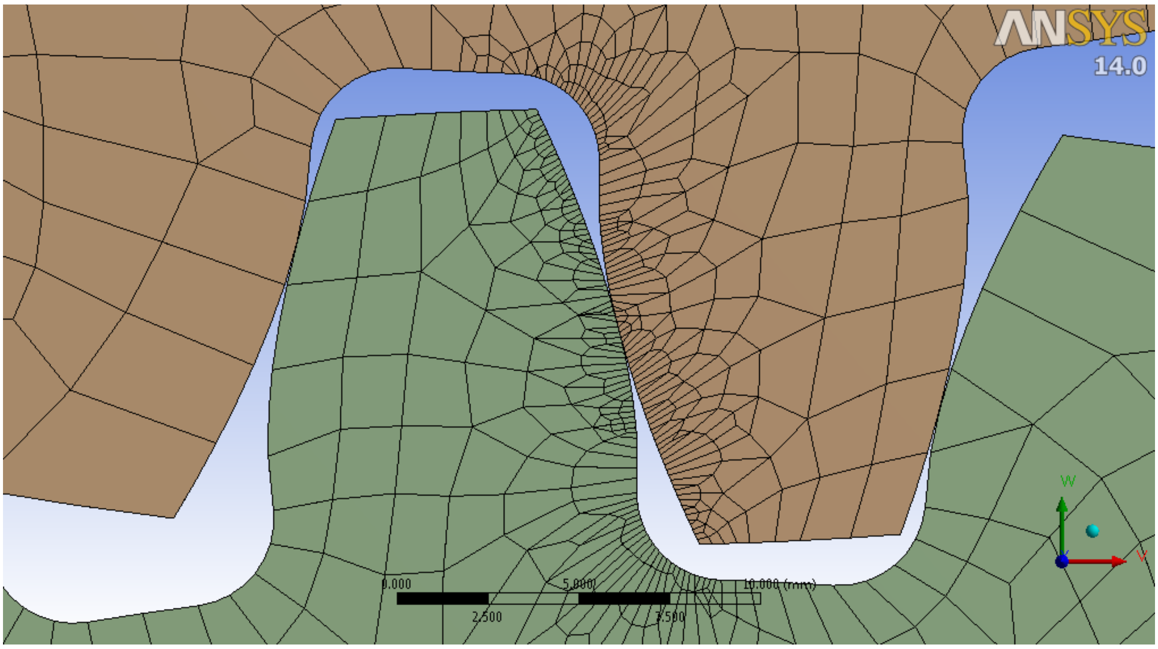

Figure 3: Meshing

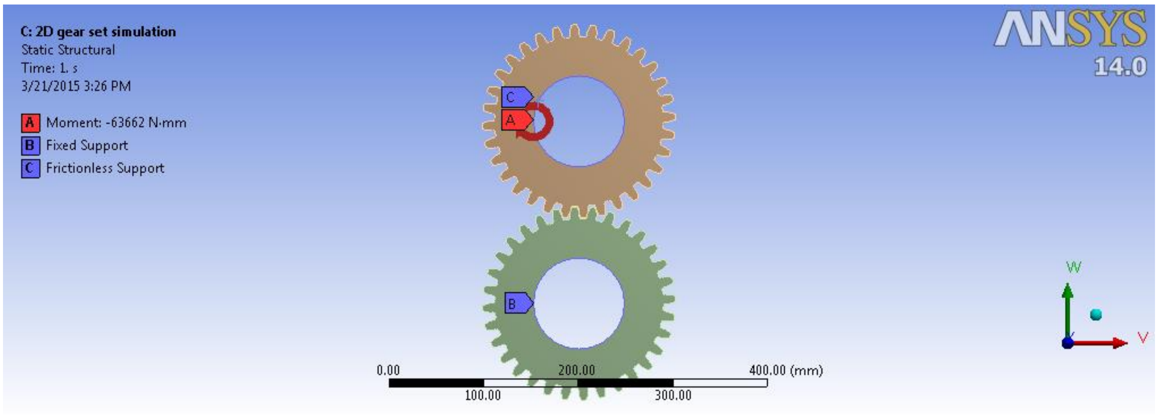

Figure 4: Boundary condition

VI. Results & Discussions:

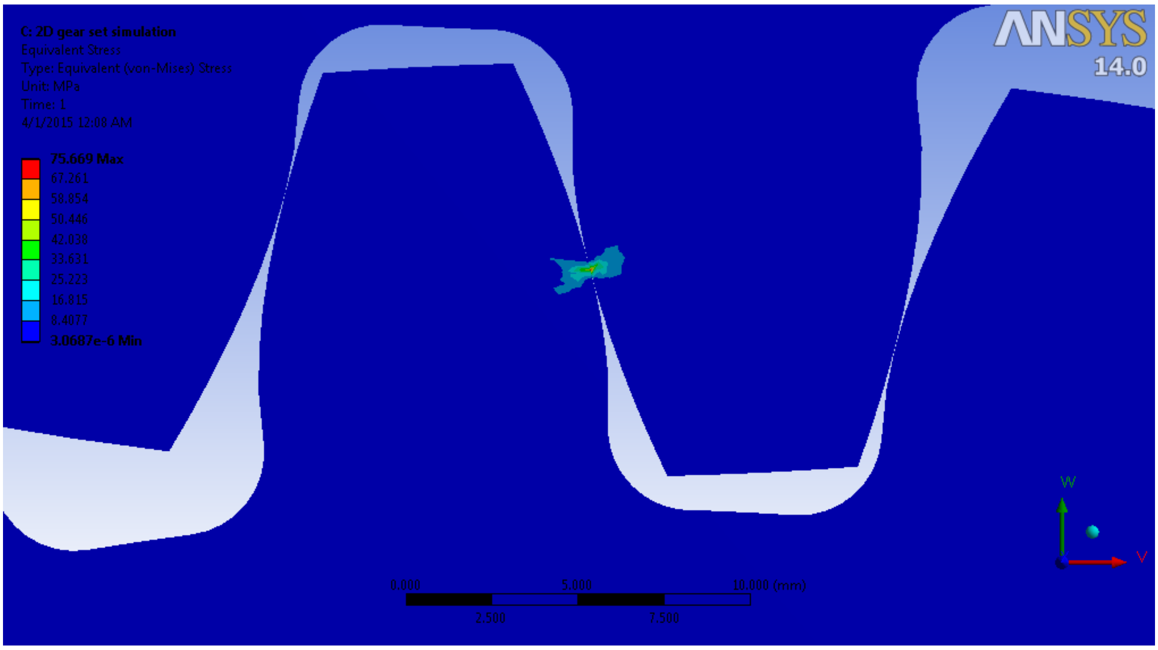

Figure 5: Equivalent Stress

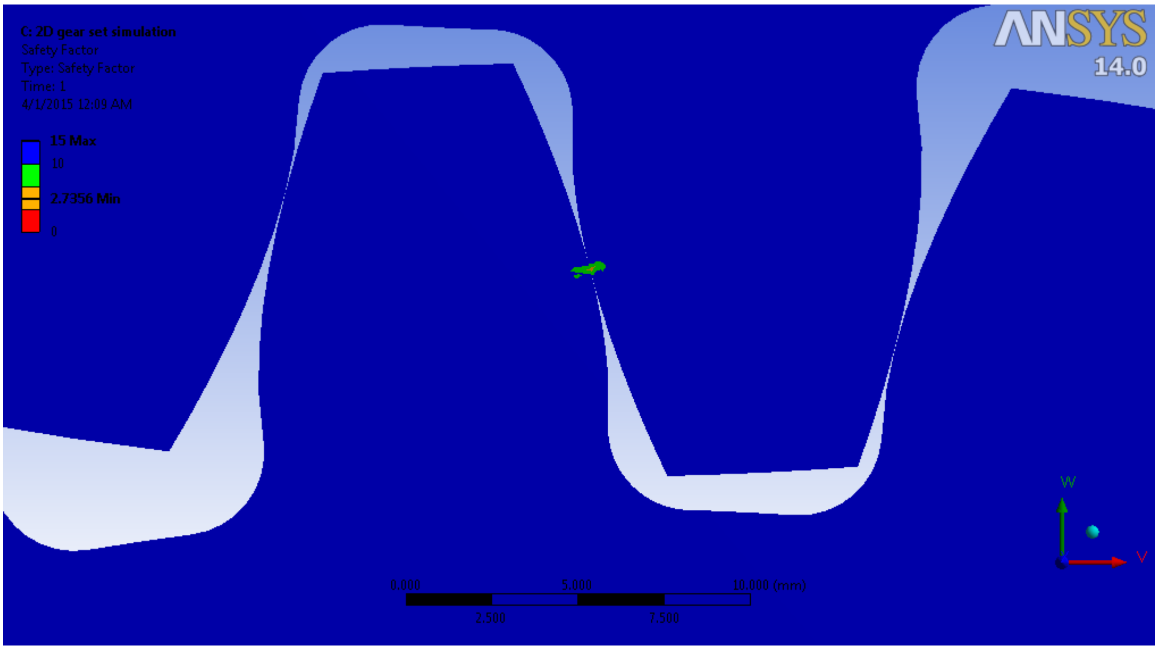

Figure 6: Safety Factor

As is clearly indicated in figure 5, the maximum allowable contact stress for stainless spur gears of dimensions given in table 2 and for transmitted torque of 63662 N-mm is 75.669MPa, as determined by FEA using ANSYS Workbench 14.0 and as obtained from Hertz equations for the same specifications in section 4 is 75.171MPa. We observe that the FEA and theoretical results are in good degree of agreement with each other with a percentage difference of just 0.66%.

VII. Conclusion

We conclude that FEA provides results that are comparable with theoretical analysis results as was in the contact stress analysis of spur gears in the present study. FEA can predict whether a product will break, wear out, or work the way it was designed. Hence, FEA can prove very helpful in the product development process by forecasting its behaviour in operation.

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